Probabilistic Taken's Embedding through the Wasserstein Tangent Space

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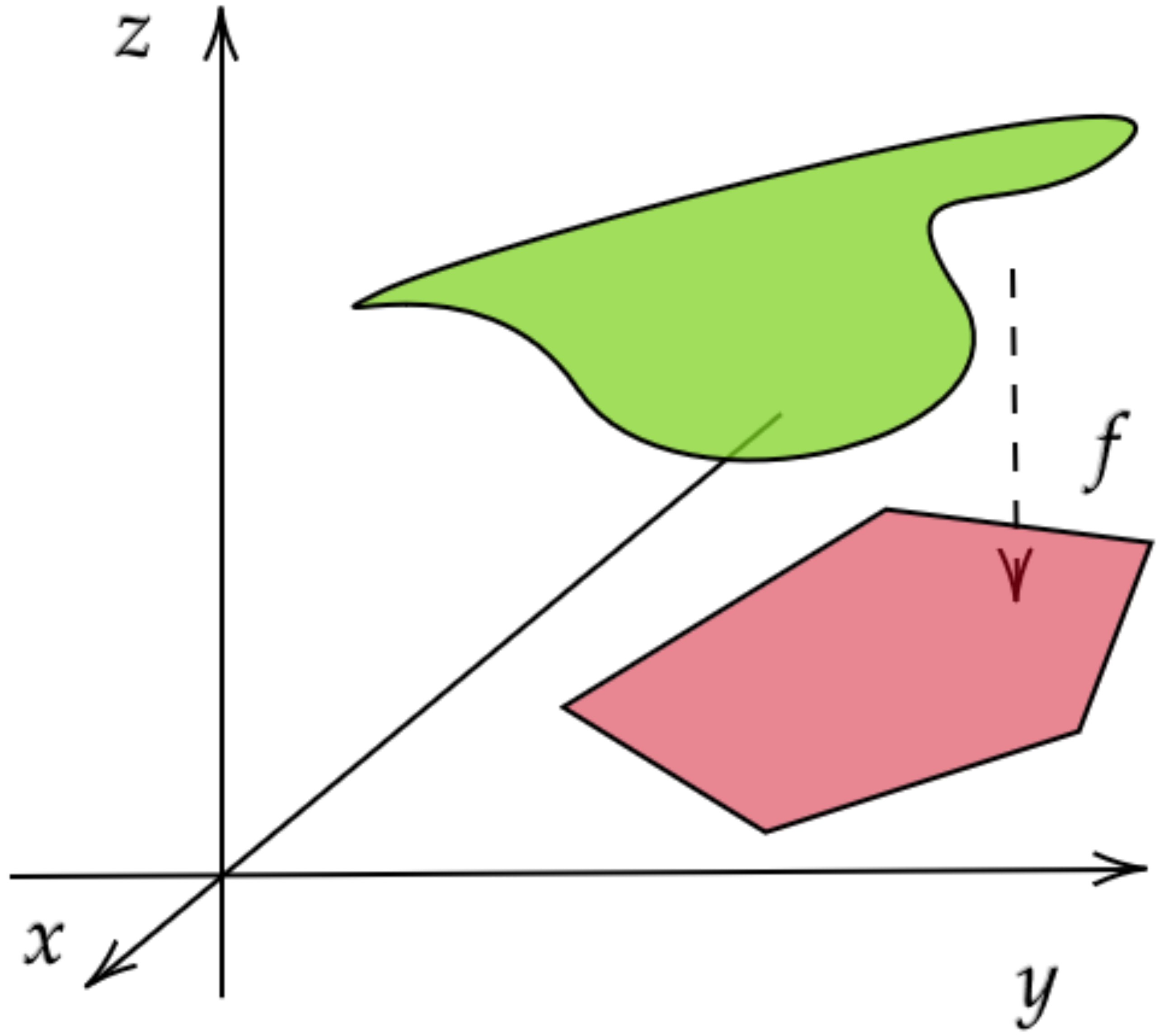
Embeddings

- Notion of **„sameness"**
- Diffeomorphism that **preserves the differential structure**
- $f : M \rightarrow N$ is an **embedding** if
  - f is **bijective** onto $f(M)$
  - f is **differentiable**
  - Df_x is **injective**



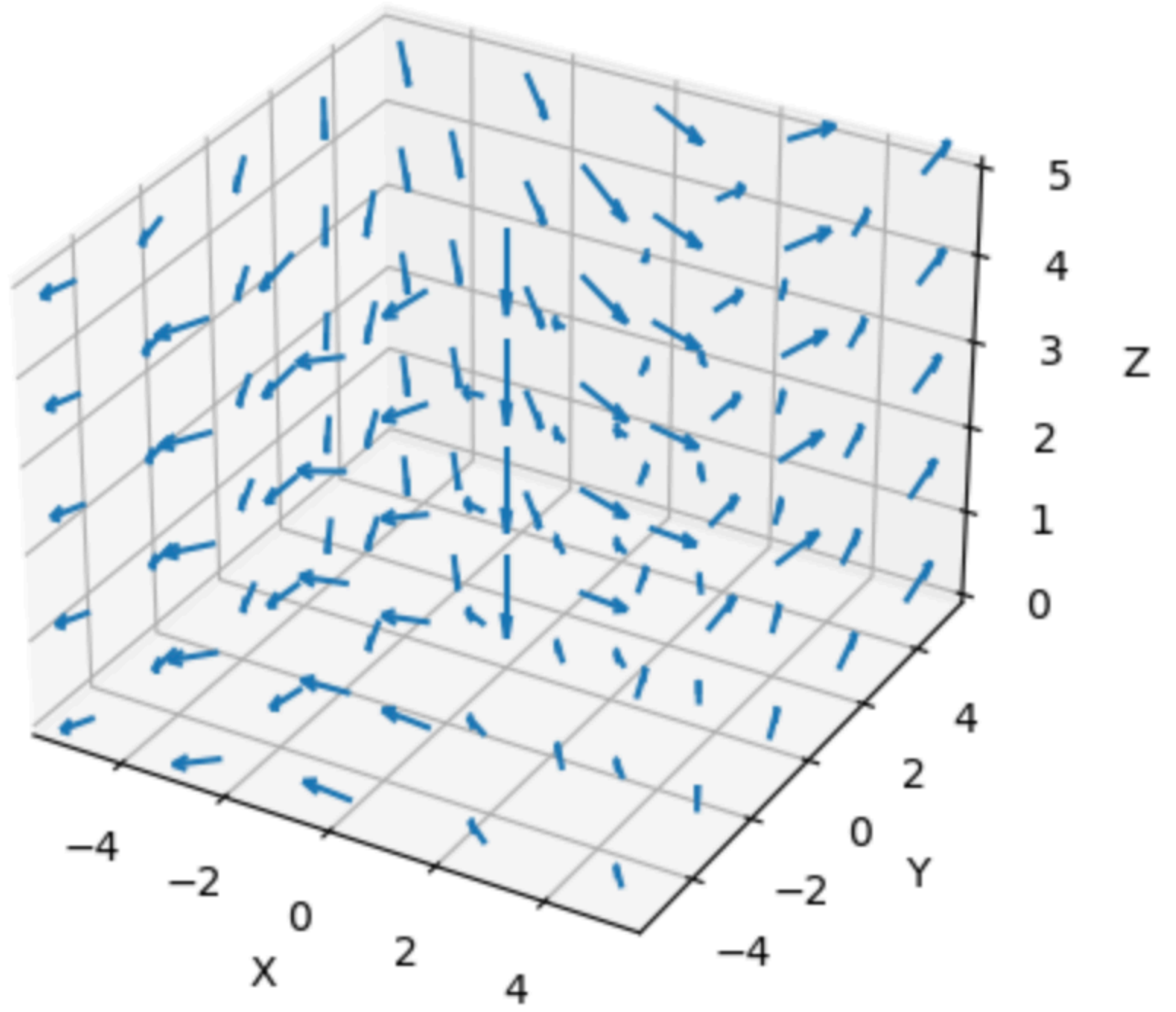
Takens embedding

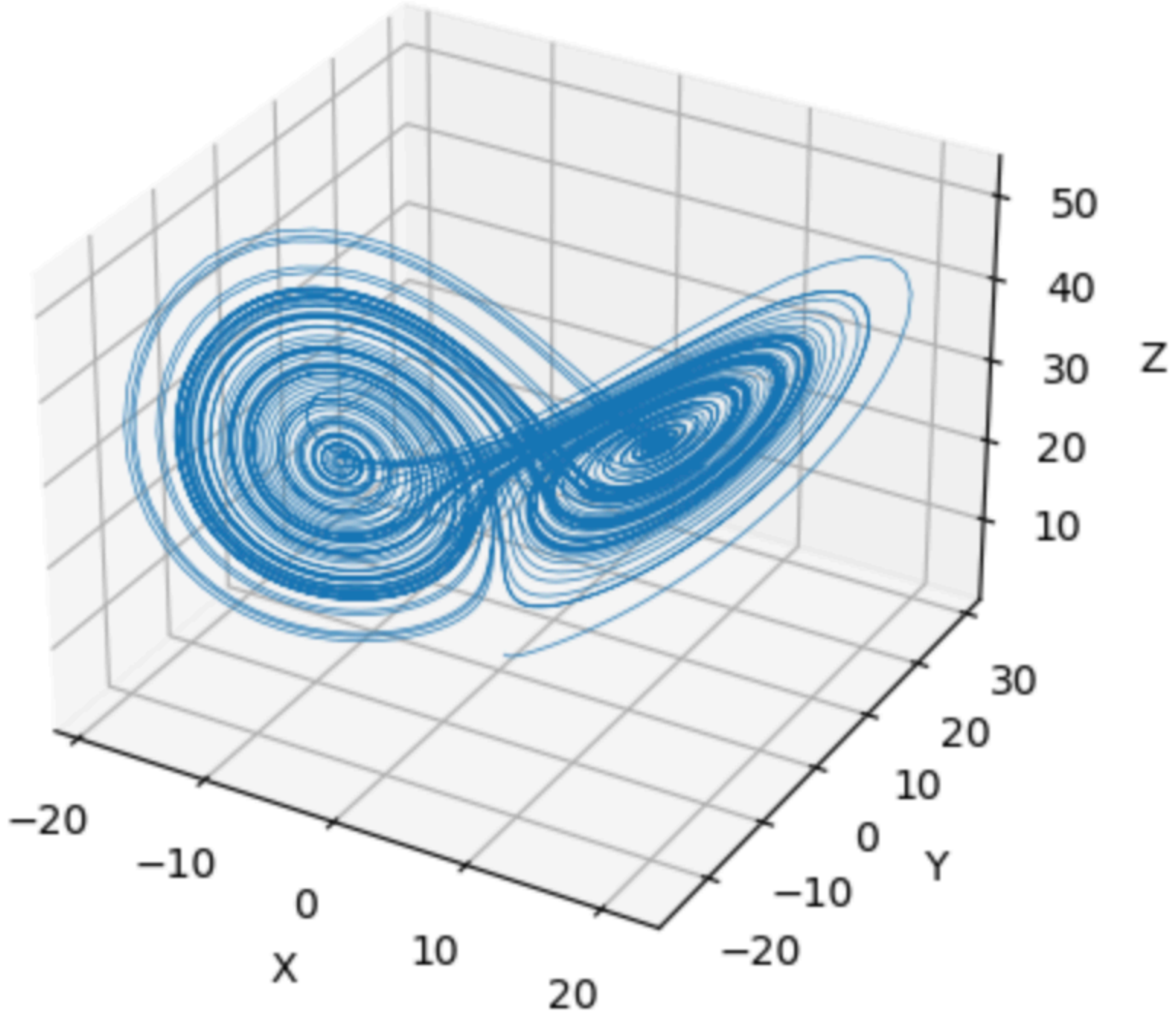
- $\dot{x} = f(x)$ with flow $\varphi_t : M \to M$
- $h : M \to \mathbb{R}$ is the **observation** function
- It is a **generic** propriety that the **delay** map

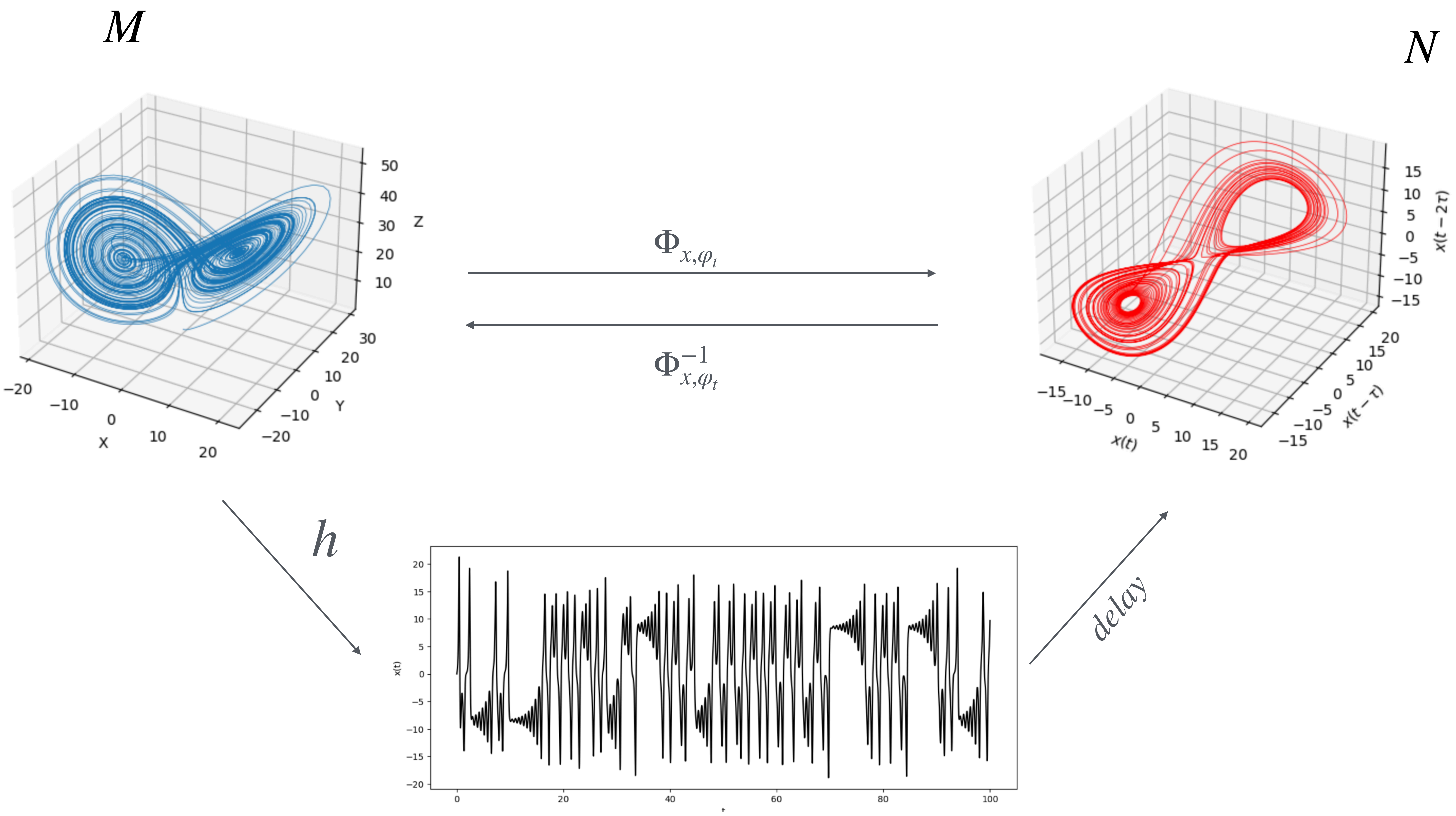
$$\Phi_{h,\varphi_t}(x) = (h(x), h(\varphi_\tau(x)), \ldots, h(\varphi_{(d-1)\tau}(x)))$$

is an **embedding**

- τ is called the delay, and d is the dimension

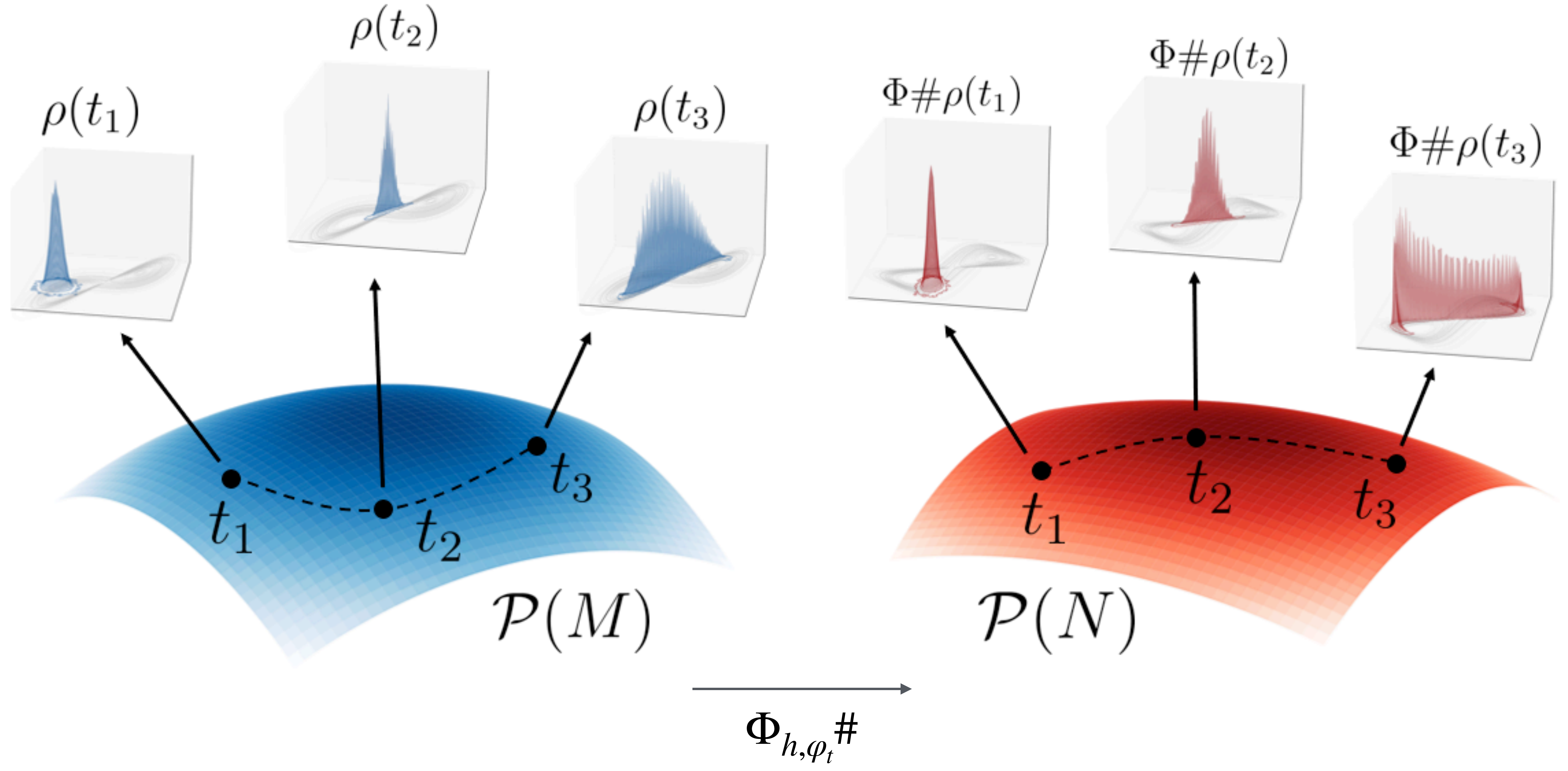




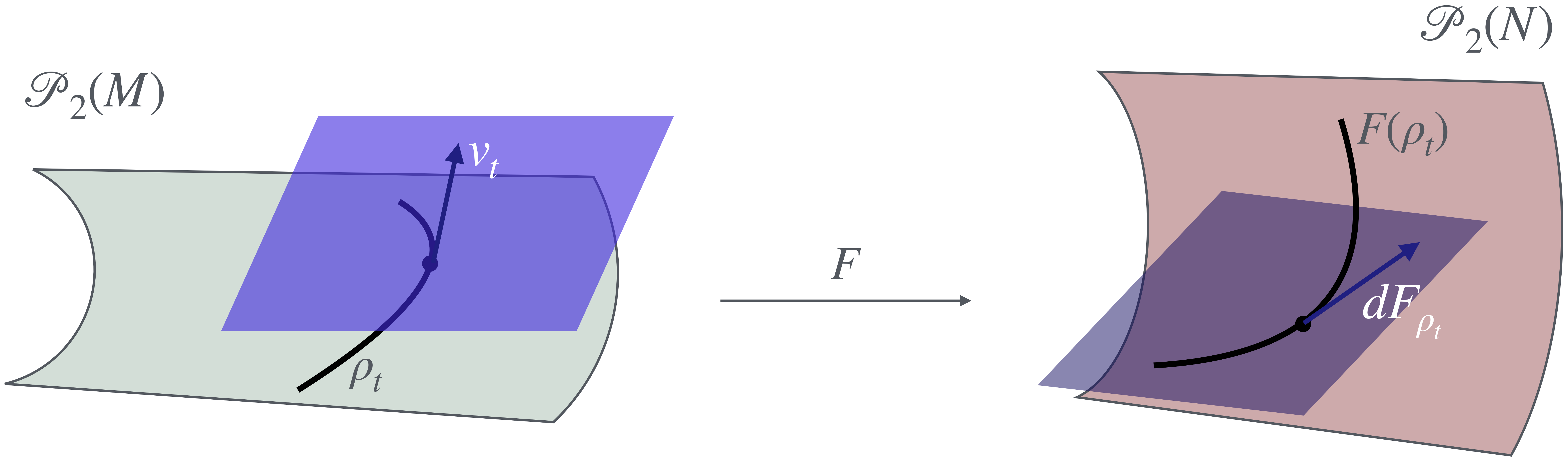


M
N
Φ_{x,φ_t}
Φ^{-1}_{x,φ_t}
h
delay
X
Y
Z
x(t)
$x(t-\tau)$
$x(t-2\tau)$

Lifted Takens embedding



Wasserstein embedding



$$\partial\rho_t + \nabla(\rho_t v_t) = 0$$

$$\partial F(\rho_t) + \nabla(F(\rho_t) dF_{\rho_t}(v_t)) = 0$$

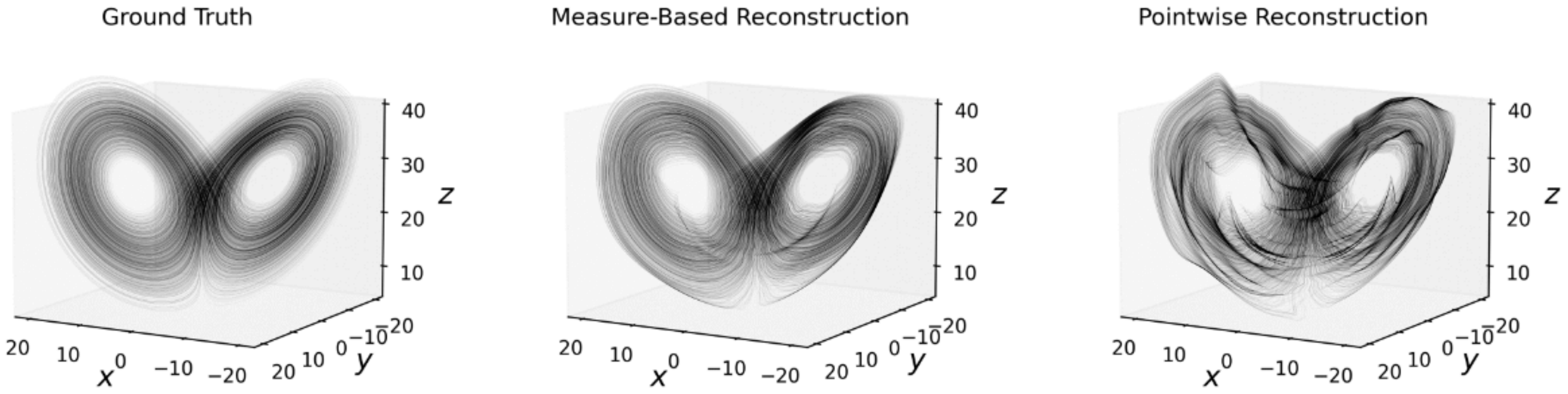
- If $F = f\#$ then $dF_{\rho_t}(v_t)(y) = P_{F(\rho_t)} df_{f^{-1}(y)} v_t(f^{-1}(y))$

Numerical result

Reconstruction map

$$L_m = \frac{1}{M}\sum_{i=1}^{M} D(\mu_i, R_\theta\#\Phi\#\mu_i)$$

$$L_{pw} = \frac{1}{N}\sum_{i=1}^{n} \|x_i - R_\theta(\Phi(x_i))\|^2$$



$0.1I$ noise